

# Advanced Designs: A Preliminary Comparison

James H. Steiger

Department of Psychology and Human Development  
Vanderbilt University

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# Introduction

- There are many advanced ANOVA designs that are appropriate for various situations you might encounter in your research.
- In this module, we examine the “building blocks” of these more complex designs, and discuss some fundamental considerations involving the cost-efficiency tradeoff that always characterizes a choice of design.

# Introduction

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- In this module, we examine the “building blocks” of these more complex designs, and discuss some fundamental considerations involving the cost-efficiency tradeoff that always characterizes a choice of design.

# The Basic Building Blocks

- Roger Kirk, in his classic book *Experimental Design: Procedures for the Behavioral Sciences*, stated that there are three *building block* designs,
  - ① *Completely Randomized* (CR)
  - ② *Randomized Block*(RB)
  - ③ *Latin Square* (LS)
- Kirk went on to state that “all complex ANOVA designs can be constructed by combining two or more of these designs.”
- So let’s start by (re-)examining these designs in the context of what we’ve learned so far.
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# The Completely Randomized Design with $p$ Levels (CR- $p$ ) Introduction

- We are, of course, already very familiar with this design.
- In the basic design, we see the following characteristics:
  - One treatment with  $p \geq 2$  treatment levels. The levels of the treatment may vary quantitatively or qualitatively. When  $p = 2$ , the design is equivalent to the two-sample, independent sample  $t$  test design.
  - Random assignment of experimental units to treatment levels, with each experimental unit receiving only one level. The number of units at each level need not be equal, but equal  $n$  is highly desirable.

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## Completely Randomized Design Layout (CR- $p$ )

- The diagram on the next slide shows the block layout for a CR-3 design.
- There are 3 groups, and  $n$  experimental units (“Subjects” in this case) are assigned randomly to each of the 3 groups.
- Each subject receives only one treatment.
- Dependent variable mean for the  $j$ th group is  $\bar{Y}_{\bullet j}$ , as shown in the diagram.



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# Completely Randomized Design Block Layout

<i>Group</i>		<i>Treatment Level</i>	
Group <sub>1</sub>	Subject <sub>1</sub>	$a_1$	$\bar{Y}_{\cdot 1}$
	Subject <sub>2</sub>	$a_1$	
	⋮	⋮	
	Subject <sub><math>n</math></sub>	$a_1$	
Group <sub>2</sub>	Subject <sub><math>n+1</math></sub>	$a_2$	$\bar{Y}_{\cdot 2}$
	Subject <sub><math>n+2</math></sub>	$a_2$	
	⋮	⋮	
	Subject <sub><math>2n</math></sub>	$a_2$	
Group <sub>3</sub>	Subject <sub><math>2n+1</math></sub>	$a_3$	$\bar{Y}_{\cdot 3}$
	Subject <sub><math>2n+2</math></sub>	$a_3$	
	⋮	⋮	
	Subject <sub><math>3n</math></sub>	$a_3$	

# Randomized Blocks Designs

## Introduction

- *Blocking* is a method for factoring out nuisance variables.
- There are two closely related randomized blocks designs:
  - *The Generalized Randomized Block Design (GRB- $p$ )*. In this version, there is more than 1 observation per cell.
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# The Generalized Randomized Blocks Design

- One way that a good experimental design achieves its purpose is by parceling out sources of extraneous error variance so that effects of interest may be detected and measured with higher precision.
- Both types of randomized blocks designs account for potential sources of irrelevant (or “nuisance”) variation by grouping experimental observations in blocks according to a score on the nuisance variable.
- Blocks are relatively homogeneous on the nuisance variable, this can be exploited to obtain a more precise estimate of the effect(s) of interest.

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# The Generalized Randomized Blocks Design

- Here is how the design works. A variable that is, relative to the question of interest, a “nuisance variable,” is isolated. Presumably, this nuisance variable has an impact on the dependent variable of interest, and so obtaining some control over it can be very useful.
- Observations are grouped in *blocks* according to their score (or likely score) on the blocking variable.
- For example, you are measuring satisfaction with several methods of instruction, but it is known that prior interest in the subject matter affects satisfaction. Your real interest is in which measure of instruction works best. One approach would be to parcel the participants into blocks according to their level of interest. For example, if you have 75 participants, you can order them from lowest to highest, in terms of their level of interest. Then you can divide them into 5 blocks, with the first block containing the 15 participants with the lowest level of interest.
- Participants within a block will be more similar in level of prior interest in the subject matter than than subjects across different blocks will be. These participants are randomly assigned to treatments. So suppose there are 3 methods of instruction, and 5 levels of interest in the subject matter as measured by a preliminary questionnaire.
- Suppose that there are 75 participants, divided equally into 5 blocks (15 participants per block). Each block of 15 participants is assigned randomly to the 3 conditions, as shown on the next slide.

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Block <sub>2</sub>	$a_1$	$a_2$	$a_3$	$\bar{Y}_{2\cdot}$
Block <sub>3</sub>	$a_1$	$a_2$	$a_3$	$\bar{Y}_{3\cdot}$
Block <sub>4</sub>	$a_1$	$a_2$	$a_3$	$\bar{Y}_{4\cdot}$
Block <sub>5</sub>	$a_1$	$a_2$	$a_3$	$\bar{Y}_{5\cdot}$
	$\bar{Y}_{\cdot 1}$	$\bar{Y}_{\cdot 2}$	$\bar{Y}_{\cdot 3}$	

- Each row has 3 cells, with  $n = 5$  participants randomly assigned to each cell.
- Each row has 15 “blocked” participants selected to be relatively homogeneous on the nuisance variable with row.

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# The Randomized Blocks Design

- The Randomized Block Design (RB- $p$ ) is simply a GRB- $p$  design with only  $p$  observations per block, and hence  $n = 1$  observations per cell.

## Two-Sample Dependent Sample $t$

- It can be conceptually confusing at first, but the randomized block design serves “double duty,” because it actually handles two fundamentally different sampling schemes.
- In one scheme, the same subject participates in several different treatments.
- In another version, the subject only receives one treatment.
- Let's see why.

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- First, we need to recall that the observations that are “blocked” can be on different people, or on the same people!
- Why? Because the key assumption of the design is that the observations within block are relatively homogeneous on the nuisance variable(s), and that they are randomly assigned to condition within block.
- To understand this in simplest terms, let’s go back to one of our earliest designs, the two-sample, correlated sample  $t$  test.
- This design can be used to handle two fundamental situations:
  - The same individuals (or experimental units) are measured twice.
  - Pairs of individuals are selected that are naturally correlated.
- Let’s diagram a correlated sample  $t$ -test as a randomized blocks design.



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Block <sub>3</sub>	$a_1$	$a_2$	$\bar{Y}_3$
⋮	⋮	⋮	⋮
Block <sub>10</sub>	$a_1$	$a_2$	$\bar{Y}_{10}$
	$\bar{Y}_1$	$\bar{Y}_2$	

- Now consider the case where Treatment Level 1 is simply Time 1, and Treatment Level 2 is simply Time 2. Then we have the classic “repeated measures” correlated sample  $t$  test.
- On the other hand, suppose the dependent measure is percent of body weight lost with a dieting plan, and the experimental “Treatment” variable is “Husbands” versus “Wives,” the purpose of the experiment being to see who loses more weight when couples diet together.
- In this case, the “block” is the couple, and there are actually two participants in each row.
- What holds for the dependent sample  $t$  test holds for randomized blocks in general, and makes them extremely versatile.
- When there is one participant per block, and that participant receives all the “treatments,” we often refer to the design as a *Repeated Measures ANOVA*.

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# Two-Sample Dependent Sample $t$

Block	Treatment Level 1	Treatment Level 2	
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Block <sub>2</sub>	$a_1$	$a_2$	$\bar{Y}_2$
Block <sub>3</sub>	$a_1$	$a_2$	$\bar{Y}_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Block <sub>10</sub>	$a_1$	$a_2$	$\bar{Y}_{10}$
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# Randomized Blocks Design

- If we look at the typical RB- $p$  design, we see that it is really a 2-way factorial design in which observations are not independent across treatments within blocks.
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Expected Mean Squares  
 CRF- $ab$  Design

Effect	A fixed B fixed	A fixed B random	A random B fixed	A random B random
A	$\sigma_e^2 + bn\theta_A^2$	$\sigma_e^2 + bn\theta_A^2 + n\sigma_{AB}^2$	$\sigma_e^2 + bn\sigma_A^2$	$\sigma_e^2 + bn\sigma_A^2 + n\sigma_{AB}^2$
B	$\sigma_e^2 + an\theta_B^2$	$\sigma_e^2 + an\sigma_B^2$	$\sigma_e^2 + an\theta_B^2 + n\sigma_{AB}^2$	$\sigma_e^2 + an\sigma_B^2 + n\sigma_{AB}^2$
AB	$\sigma_e^2 + n\theta_{AB}^2$	$\sigma_e^2 + n\sigma_{AB}^2$	$\sigma_e^2 + n\sigma_{AB}^2$	$\sigma_e^2 + n\sigma_{AB}^2$
S/AB	$\sigma_e^2$	$\sigma_e^2$	$\sigma_e^2$	$\sigma_e^2$

- Recall that  $MS_{S/AB}$  has an expected value of  $\sigma_e^2$ , but it cannot be computed if  $n = 1$  per cell.
- Suppose the Treatment variable is  $A$ , and the blocking variable  $B$ . How would we test the treatment effect if the blocking variable is a random effect?
- How would you test  $A$  if  $B$  is a fixed effect?
- Suppose you were to assume that the model is additive, i.e.

# The Latin Square Design

- This design is set up to allow an experimenter to control for the effects of two nuisance variables simultaneously.
- It is a purely additive model — it assumes that none of the effects interact.
- The design requires that the number of levels of each nuisance variable be equal to the number of treatments.



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## LS-3 Design

	$c_1$	$c_2$	$c_3$
$b_1$	$a_1$	$a_2$	$a_3$
$b_2$	$a_2$	$a_3$	$a_1$
$b_3$	$a_3$	$a_1$	$a_2$